VARIATIONAL METHOD APPLIED TO FORMANTS COMPUTATION FOR A PHARYNGO-BUCO-NASAL TRACT

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Abstract

In this paper, we show how the variational method can be used in order to compute the formants and the sensitivity functions related to a vocal tract when the velum is open. The stationary modes of vibration obtain by this method (whithout the need to compute the transfer function) lead to a physical description of anti-formants.

1 Introduction

The variational formulation of the acoustico-articulatory link has already been successfully applied to a single vocal tract(disregarding the nasal tract)[1]. It provides us with a quick numerical algorithm allowing us to compute stationary vibration frequencies(the formants) for a given vocal tract whose geometry is represented by an area function.

This method may be extended to the case where the nasal tract is connected by imposing both flow conservation and pressure continuity at the velum.

2 Acoustic model

2.1 The Sondhi Model

We have adopted the acoustic modeling of the vocal tract proposed by Sondhi[2]. This model is characterized by a parietal admittance proportional to the area function, a constant shape factor, and a sound propagation in quasi planar wave fronts.

Let us consider the velocity potential $\Phi(x,t)$ where x is the distance from the glottis and t is the time. The volume velocity ν and the average pressure p are given by:

$$\nu(x,t) = -\Phi_{,x} \ ; \ p(x,t) = \rho\Phi_{,t} \qquad (1)$$

Henceforth,we will note either $\partial_x g$ or $g_{,x}$ for the partial derivative $\frac{\partial g}{\partial x}$ of function g. For a normal mode at frequency $f = \omega/2\pi$, the velocity potential may be written:

$$\Phi(x,t) = \Psi(x)e^{-\sigma t}cos(\omega t), \qquad (2)$$

where Ψ is the spatially distributed mode amplitude and σ represents the damping of the field caused by wall elasticity. In such a case, it can easily be demonstrated that $\Psi(x)$ verifies the following Webster equation:

$$\partial_x(A(x)\partial_x\Psi(x)) + \frac{(\omega^2-\omega_p^2)}{c^2}A(x)\Psi(x)=0 \qquad (3)$$

where ω_p is the resonance frequency of walls ($\sim$ 200 $\times$ $2\pi s^{-1}$), and c, the sound velocity ($c = 34000 cm/s$). For homogenous boundary conditions, this equation has solutions only for discrete values of ω.

2.2 Tract ends conditions

In this paper, we will consider the case where the glottis is closed and assume an infinite glottis impedance. Hence, we have:

$$\partial_x\Psi(0) = 0. \qquad (4)$$

Considering that the lip radiation can be approached by that of a vibrating piston set in a spherical baffle(the head) [1] leads us to :

$$A(L)\Psi_{,x}(L) + q\sqrt{A(L)}\Psi(L) = 0 \quad (5)$$

with L being the total tract length(i.e the distance between the glottis and the lips), and q a factor depending on the degree of aperture of the lips. As a first empirical estimation, we propose:

$$q = a\sqrt{A(L)} + b \qquad (6)$$

with $a = -3.5cm^{-1}$ and $b = 35.0$

2.3 Three tracts linked together

In this case, we will need three velocity potential functions in order to describe the wave behavior in a three tracts system. Let Ψ_{ph},Ψ_b and Ψ_n be those functions for respectively pharyngal, bucal and nasal tracts.

For the sake of simplicity we change the variable x into z such as:

$$z_\alpha = x_\alpha/L_\alpha \qquad (7)$$

with α taken as ph, b or n. We have thus, for each tube the related Webster equation (4) which is now written:

$$\partial_z(A_\alpha\partial_z\Psi_\alpha)+((\omega^2-\omega_{p,\alpha}^2)\frac{L_\alpha^2}{c^2})A_\alpha\Psi_\alpha=0 \qquad (8)$$

We emphasize in eq (8) that ω doesn't take an symbol α (ph, b or n),it represents the angular resonance frequency of the whole three tubes system.These are coupled together by imposing both pressure continuity:

$$\Psi_{ph}(1) = \Psi_b(0) = \Psi_n(0) \qquad (9)$$

and flow conservation:

$$\frac{A_{ph}(1)}{L_{ph}}\partial_z\Psi_{ph}(1) - \frac{A_b(0)}{L_b}\partial_z\Psi_b(0) - \frac{A_n(0)}{L_n}\partial_z\Psi_n(0) = 0 \ . \qquad (10)$$

The glottis condition (4) is

$$\partial_z\Psi_{ph}(0) = 0, \qquad (11)$$

and the lip and nostril conditions (5) become (by 7):

$$A_b(L_b)\partial_z\Psi_b(L_b)+q\,L_b\sqrt{A_b(L_b)}=0 \quad (12.1)$$

$$A_b(L_n)\partial_z\Psi_n(L_n)+q\,L_n\sqrt{A_n(L_n)}=0 \quad (12.2)$$

3 Variational method

Let

$$I=\int_{z_0}^{z_1} \mathcal{L}(z,\Psi(z),\Psi'(z))dz + G(\Psi(z_0),\Psi(z_1)) \quad (13)$$

The extremals of I (here, minimal) for which $\delta I = 0 \ \forall\delta\Psi \ll \Psi$ verify the system:[3]

$$\frac{\partial\mathcal{L}}{\partial\Psi} - \frac{d}{dz}\frac{\partial\mathcal{L}}{\partial\Psi'} = 0 \qquad (14.1)$$

$$\frac{\partial\mathcal{L}}{\partial\Psi'}|_{z_1} + \frac{\partial G}{\partial\Psi(z_1)} = 0 \qquad (14.2)$$

$$\frac{\partial\mathcal{L}}{\partial\Psi'}|_{z_0} - \frac{\partial G}{\partial\Psi(z_0)} = 0 \qquad (14.3)$$

We will now build three functionals(I_α, α = ph, b, n) whose extremals are solutions of Webster equations (8) with respect to the junction conditions (9) and (10) and the boundaries conditions (11) and (12).

Let

$$\mathcal{L}_\alpha(z,\Psi_\alpha,\Psi_\alpha')=\frac{1}{L_\alpha}A_\alpha(z)((\partial_z\Psi_\alpha(z))^2 - \frac{L_\alpha^2}{c^2}(\omega^2-\omega_{p,\alpha}^2)\Psi_\alpha(z)^2) \quad (15)$$

α taken as ph, b or n and

$$G_{ph}=-2\Psi_{ph}(1)[\frac{A_b(0)}{L_b}\partial_z\Psi_b(0)+\frac{A_n(0)}{L_n}\partial_z\Psi_n(0)] \quad (16)$$

$$G_b=2\Psi_b(0)[\frac{A_{ph}(1)}{L_{ph}}\partial_z\Psi_{ph}(1)-\frac{A_n(0)}{L_n}\partial_z\Psi_n(0)] +\Psi_b(0)[2\Psi_{ph}(1)-\Psi_b(0)]+q\sqrt{A_b(1)}L_b\Psi_b^2(1)$$

($G_n = G_b$ transposing b and n) be the $\mathcal{L}$ and G functions for the three functionals. The extremals of those functionals I_α are solutions of the following variational system:

$$\delta I_\alpha/\delta\Psi_\alpha = 0, (\alpha = ph, b, n) \quad (17)$$

and thus verify the system (14).

Now,it is easy to show that the equation (14.1)→(14.3) applied to (15) & (16) gives us the wanted Webster equations (8), pressure continuity (9), flow conservation (10) and the glottis, lip and nostril conditions (11) and (12).

4 Resonance mode computation

The extension of the Rayleigh-Ritz[4] method in the case where three functionals have to be minimized together will lead to a fast and precise algorithm allowing us to compute resonance modes frequencies and the associated wave functions .

Let η_i be a set of n linearly independent functions(we have chosen here Tchebychev polynoms). Suppose that three functions Ψ_{ph},Ψ_b and Ψ_n verify (17) and so are solutions of equations (8)→(12).These functions may be written as an approximation such as

$$\Psi_{ph}(z) = \sum_{i=1}^{n} C_i\eta_i(z) ; \quad \Psi_b(z) = \sum_{i=n+1}^{2n} C_i\eta_{i-n}(z) \quad \Psi_n(z) = \sum_{i=2n+1}^{3n} C_i\eta_{i-2n}(z) \quad (18)$$

where,here for the sake of simplicity, we have taken the same number of approximation functions for each of the three tubes.

Injecting (18) into (16) we can write

$$I_\alpha = I_\alpha(C_1, ..., C_{3n}, \eta_1, ..., \eta_n, \eta_1', ..., \eta_n') \quad (19)$$

for α as ph,b or n.

Looking for the extremals of these three functionals,we will impose:

$$\frac{\partial I_{ph}}{\partial C_i} = 0 \quad \forall i \in (1, ..., n) \quad \frac{\partial I_b}{\partial C_i} = 0 \quad \forall i \in (n+1, ..., 2n) \quad \frac{\partial I_n}{\partial C_i} = 0 \quad \forall i \in (2n+1, ..., 3n) \quad (20)$$

after having replaced the Ψ_α by their developments (18).

Thus we obtain three systems of n equations and $3n$ unknowns coupled together thanks to the junction terms and giving one system of $3n$ equations and $3n$ unknowns of the form

$$\sum_{i=1}^{3n}(V_{ij} - \omega^2 W_{ij})C_j = 0 \quad (21)$$

for $i \in (1, .., 3n)$ which can be written in matrix form.

Multiplying on the left by W_{ij}^{-1},we obtain

$$(W^{-1}V - \omega^2\mathbb{1})\overline{C} = \overline{0} \quad (22)$$

$\mathbb{1}$ being the unitary matrix $\mathbb{R}^{3n} \to \mathbb{R}^{3n}$.

(22) has non trivial solutions if and only if ω^2 is eigen value of the operator

$W^{-1}V$. So the first s $(\omega_1, ..., \omega_s)$ eigenvalues(associated to the formants) and corresponding eigen vectors (the $\overline{C}_s$ giving the solutions (18)) can be computed easily by classical numerical methods.

Given the stationarity of the functional for the l^{th} mode $\Psi_{\alpha,l}$, the sensitivity functions can be obtained from the conditions:

$$I_\alpha(\omega_l + \delta\omega_l, A(x) + \delta A(x), \Psi_{\alpha,l}, \partial_x\Psi_{\alpha,l}) = I_\alpha(\omega_l, A(x), \Psi_{\alpha,l}, \partial_x\Psi_{\alpha,l}) \quad (23)$$

where $\delta A(x)$is an area function perturbation localised at x. These conditions are correct at the first order of perturbations.

4.1 Results

We will briefly discuss our results using a simple geometrical configuration. Considering a system of three uniform tracts, such as:
$L_{ph} = L_b = 8.5cm, L_n = 9cm$, $A_{ph}(0) = 0, A_{ph}(x) = 5cm^2$ and $A_b(x) = A_n(x) = 2.5cm^2$, fig 1 shows two stationary modes extracted from the set obtained.

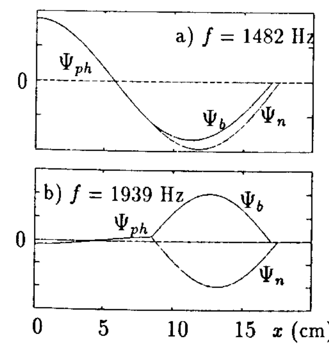

fig 1:wave function amplitude for two stationary modes

For a given resonance frequency, the pressure as function of time must be seen as the the projection of wave function Ψ rotating at angular frequency $\omega = 2\pi f$ around the x axis. The first (fig 1.a) shows a passing resonance,the acoustic impulsion($A\partial_x\Psi$) outgoing from pharynx being shared between the nasal and bucal tracts.

In fig 1.b , both potential($\div\Psi^2$) and kinetic($\div\partial_x\Psi^2$) energy are low at the end of the pharynx. No significant signal will cross the junction if the source is at the glottis. We think that such a normal mode will lead to an anti-formant and we intend to test our predictions on an experimental device. We can already say that, for some realistic geometrical configurations, we observe both a lowering of the first formant F1 and the occurance of a nasal formant between F1 and F2 which are characteristics of nasalised vowels described in the literature[5].

References

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